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Supergravity solution for three-string junction in M-theory

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Abstract: Three-string junctions are allowed configurations in IIB string theory which preserve one-fourth supersymmetry. We obtain the 11-dimensional supergravity solution for curved membranes corresponding to these three-string junctions.

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In the last few years, there has been lot of interest on three-string junctions in type-IIB string theory [1]–[7]. It was originally postulated by Schwarz that string theory can also admit multi-junction/multi-pronged strings as fundamental objects [1]. In ref. [2], it has been shown that the gauge field configurations in F-theory on K_3 for exceptional groups can be accounted only if multi-pronged strings besides the usual strings (two-pronged) are included as solutions in string theory. The one-fourth BPS nature of three-string junction was conjectured in [1] and proven in [3, 4]. These three-pronged strings/three-string junction connecting three D3-branes gives understanding of $\frac{1}{4}$th BPS states of the SU(3) super Yang-Mills theory on the coincident three D3-branes [5, 6] and their SU(N) generalisations presented in ref. [7].

With enough evidence for such a junction configuration to be present in string theory, it is very essential that these junctions emerge as exact solution of supergravity field equations. Till date, attempts in 10 dimensions have failed due to the singular nature at the junction. The hope is that smoothening the junction would help in finding the solution. Hence, we look at curved membranes in M-theory corresponding to three-string junctions in IIB theory. The supergravity solution for planar membranes corresponding to fundamental strings [8] gives us some idea in solving the curved case. The solution for such curved membranes representing three-string junction in IIB theory, after the forthcoming (tedious) calculations, are given in eqs. (61) and (62).[1] It is important to stress that the mathematical limitations (analytic solution for a non-linear differential equation) enables us to obtain only an implicit supergravity solution for three-string junction.

The two dimensional membrane corresponding to three-string junction is given by holomorphic curve [9]:

$$f(u,v) = \sqrt{(u-\lambda_1)(v-\lambda_2)} - \sqrt{\lambda_1\lambda_2} = 0\,, \tag{1}$$

where u and v are functions of complex coordinates U and V in $\mathbb{R}^2 \times \mathbb{T}^2$.

We begin by making an ansatz for the 11-dimensional metric g_{MN} respecting SO(6) symmetry and gauge fields A_{MNP} ($M,N,P = 0,1,\ldots 10$)

$$\begin{aligned} ds^2 &= -a_0(u,\bar u,v,\bar v,r)\,dt^2 + 2\,g_{\mu\bar\nu}(u,\bar u,v,\bar v,r)\,dx^\mu\,dx^{\bar\nu} + \\ &\quad + a_1(u,\bar u,v,\bar v,r)\sum_{i=5}^{10}(dx^m)^2\,, \qquad (2)\\ A &= A_{0\mu\bar\nu}(u,\bar u,v,\bar v,r)\,dx^0\wedge dx^\mu\wedge\wedge dx^{\bar\nu}\,, \qquad (3)\end{aligned}$$

where $r^2 = \sum_{m=5}^{10}(x^m)^2$ and $x^0 = t$, $x^\mu = u,v$; $x^{\bar\nu} = \bar u,\bar v$.

[1] We came across a recent paper [12] where a similar form appeared for localised orthogonal intersecting membranes.

The vielbeins $e^{\hat a}_M$ and inverse vielbeins $E^M_{\hat a}$ for the metric ansatz in the upper triangular form:

$$
\begin{aligned}
e^{\hat 0}_0 &= \sqrt{a_0} = \left(E^0_{\hat 0}\right)^{-1}, \qquad e^{\hat m}_n = \delta_{mn}\sqrt{a_1} = (E^m_{\hat n})^{-1}, \\
e^{\hat v}_v &= e^{\hat{\bar v}}_{\bar v} = \sqrt{g_{v\bar v}} = (E^v_{\hat v})^{-1} = \left(E^{\bar v}_{\hat{\bar v}}\right)^{-1}, \\
e^{\hat u}_u &= e^{\hat{\bar u}}_{\bar u} = \sqrt{\frac{g_{u\bar u}\, g_{v\bar v} - g_{u\bar v}\, g_{v\bar u}}{g_{v\bar v}}} = (E^u_{\hat u})^{-1} = \left(E^{\bar u}_{\hat{\bar u}}\right)^{-1}, \\
e^{\hat v}_u &= \frac{g_{\bar v u}}{\sqrt{g_{v\bar v}}}; \qquad e^{\hat{\bar v}}_{\bar u} = \frac{g_{\bar u v}}{\sqrt{g_{v\bar v}}}, \\
E^v_{\hat u} &= -\frac{g_{u\bar v}}{\sqrt{g_{v\bar v}(g_{u\bar u}\, g_{v\bar v} - g_{u\bar v}\, g_{v\bar u})}}, \qquad E^{\bar v}_{\hat{\bar u}} = -\frac{g_{v\bar u}}{\sqrt{g_{v\bar v}(g_{u\bar u}\, g_{v\bar v} - g_{u\bar v}\, g_{v\bar u})}},
\end{aligned} \tag{4}
$$

where the indices with hat refers to tangent space index distinguishing them from world volume indices. The arbitrary functions a_i's and three-form components must be reduced to fewer number of unknowns by requiring that the field configuration (2) and (3) preserve one-fourth supersymmetry. In other words, there must exist Killing spinors satisfying

$$D_M \epsilon = 0\,, \tag{5}$$

where D_M is the supercovariant derivative appearing in the gravitino supersymmetry transformation,

$$\delta\psi_M = D_M \epsilon\,, \tag{6}$$

$$D_M = \partial_M + \frac{1}{4}\,\omega_M^{\hat A\hat B}\,\Gamma_{\hat A\hat B} - \frac{1}{288}\left(\Gamma_M^{PQRS} + 8\,\Gamma^{PQR}\,\delta^S_M\right) F_{PQRS}\,, \tag{7}$$

where $F_{MNPQ} = 4\,\partial_{[M}A_{NPQ]}$. Here $\Gamma_{\hat A}$ are the $D = 11$ Dirac matrices obeying

$$\{\Gamma_{\hat A}, \Gamma_{\hat B}\} = 2\,\eta_{\hat A\hat B}\,, \tag{8}$$

where $\eta_{\hat A\hat B} = \mathrm{diag}\,(-,+,+\cdots+)$ and

$$\Gamma_{\hat A\hat B\cdots\hat C} = \Gamma_{[\hat A}\,\Gamma_{\hat B}\cdots\Gamma_{\hat C]}\,, \tag{9}$$

and Γ's with world volume index can be converted to tangent space index using vielbeins.

We will split the 11-dimensional Γ matrix respecting SO(6) symmetry in the following way:

$$\Gamma_{\hat A} = (i\,\gamma_{\hat\alpha}\otimes\Gamma_7, 1\otimes\Sigma_{\hat a})\,, \tag{10}$$

where $\gamma_{\hat\alpha}$ and $\Sigma_{\hat a}$ are the $D = 5$ and $D = 6$ Dirac matrices and

$$\Gamma_7 = \Sigma_{\hat 5}\,\Sigma_{\hat 6}\cdots\Sigma_{\hat{10}} \tag{11}$$

satisfying the following properties:

$$\gamma_{\hat 0}\,\gamma_{\hat u\,\hat{\bar u}}\,\gamma_{\hat v\hat{\bar v}} = i\,; \qquad \Gamma_7^2 = -1\,. \tag{12}$$

To simplify the computations, we shall first impose the following constraints on ϵ:

$$\gamma_u\,\epsilon = 0\,; \qquad \gamma_v\,\epsilon = 0\,; \qquad \Gamma_7\,\epsilon = -i\epsilon\,, \tag{13}$$

which in the 11 dimensions gives one-fourth BPS nature of the curved membranes corresponding to three-string junction-viz.,

$$\Gamma_1\,\Gamma_2\,\Gamma_5\cdots\Gamma_{10}\,\epsilon = \epsilon\,, \tag{14}$$

$$\Gamma_3\,\Gamma_4\,\Gamma_5\cdots\Gamma_{10}\,\epsilon = \epsilon\,. \tag{15}$$

With the above constraints, the spin connection on the spinor field can be simplified to

$$(\omega_M)_{\hat{A}\hat{B}}\,\Gamma^{\hat{A}\hat{B}}\,\epsilon = E^N_{\hat{A}}\,E^R_{\hat{B}}\,\partial_R\,g_{NM}\,\Gamma^{\hat{A}\hat{B}}\,\epsilon\,. \tag{16}$$

In our background (2) and (3), we shall now examine eq. (5). After incorporating one-fourth BPS condition (13), we get

$$\begin{aligned} D_0\,\epsilon = \partial_0\,\epsilon + \Bigg[&\frac{1}{4}\,\gamma_0\,\gamma^u(\partial_u \ln a_0) + \frac{1}{4}\,\gamma_0\,\gamma^v(\partial_v \ln a_0) - \frac{1}{4}\,\gamma_0\,\Sigma^m(\partial_m \ln a_0) - \\ &- \frac{i}{6}\,\sqrt{a_0^{-1}}\,\gamma_0\,\Sigma^m H + \frac{i}{6}\,\sqrt{a_0^{-1}}\,\gamma_0\,I\gamma^u + \frac{i}{6}\sqrt{a_0^{-1}}\,\gamma_0\,J\gamma^v\Bigg]\,\epsilon = 0\,, \end{aligned} \tag{17}$$

$$\begin{aligned} D_m\,\epsilon = \partial_m\,\epsilon + \Bigg[&\frac{1}{8}\,(\Sigma_m\,\Sigma^n - \Sigma^n\Sigma_m)(\partial_n \ln a_1) + \frac{1}{4}\,\Sigma_m\,\gamma^u(\partial_u \ln a_1) + \\ &+ \frac{1}{4}\,\Sigma_m\,\gamma^u(\partial_u \ln a_1) - \frac{i}{24}\,\sqrt{a_0^{-1}}\,(\Sigma_m\,\Sigma^n - \Sigma^n\,\Sigma_m)H - \\ &- \frac{i}{12}\,\sqrt{a_0^{-1}}\,\Sigma_m\,\gamma^u I - \frac{i}{12}\,\sqrt{a_0^{-1}}\,\Sigma_m\,\gamma^v J + \frac{i}{6}\,\sqrt{a_0^{-1}}\,H\Bigg]\,\epsilon = 0\,, \end{aligned} \tag{18}$$

$$\begin{aligned} D_u\,\epsilon = \partial_u\,\epsilon + \Bigg[&\frac{1}{4}\Big(g^{u\bar{u}}\partial_u\,g_{u\bar{u}} + g^{v\bar{v}}\partial_v\,g_{\bar{v}u} + g^{\bar{u}v}\partial_v\,g_{u\bar{u}} + \\ &+ g^{u\bar{v}}\partial_u\,g_{\bar{v}u}\Big) + \frac{i\,\sqrt{a_0^{-1}}}{12}\,I\Bigg]\,\epsilon = 0\,, \end{aligned} \tag{19}$$

$$\begin{aligned} D_v\,\epsilon = \partial_v\,\epsilon + \Bigg[&\frac{1}{4}\Big(g^{v\bar{v}}\partial_v\,g_{v\bar{v}} + g^{u\bar{v}}\partial_u\,g_{\bar{v}v} + g^{\bar{u}u}\partial_u\,g_{v\bar{u}} + \\ &+ g^{v\bar{u}}\partial_v\,g_{\bar{u}v}\Big) + \frac{i\,\sqrt{a_0^{-1}}}{12}\,J\Bigg]\,\epsilon = 0\,, \end{aligned} \tag{20}$$

$$\begin{aligned} D_{\bar{u}}\,\epsilon = \partial_{\bar{u}}\,\epsilon - \Bigg[&\frac{1}{4}\Big(g^{u\bar{u}}\partial_{\bar{u}}\,g_{u\bar{u}} + g^{u\bar{v}}\partial_{\bar{v}}\,g_{u\bar{u}} + g^{v\bar{v}}\partial_{\bar{v}}\,g_{v\bar{u}} + g^{v\bar{u}}\partial_{\bar{u}}\,g_{v\bar{u}}\Big) - \frac{1}{4}\,\Sigma^m\gamma^v\partial_m\,g_{v\bar{u}} - \\ &- \frac{1}{4}\,\Sigma^m\gamma^u\partial_m\,g_{u\bar{u}} + \frac{1}{4}\,\gamma^{uv}\,(\partial_v\,g_{u\bar{u}} - \partial_u\,g_{v\bar{u}}) - \\ &- \Sigma^m\gamma^v\frac{i}{12}\,\sqrt{a_0^{-1}}\,\Big(g^{\bar{v}u}P - g^{u\bar{u}}Q\Big)\,\Sigma^m\gamma^u\frac{i}{12}\,\sqrt{a_0^{-1}}\,\Big(g^{\bar{v}v}P - g^{v\bar{u}}Q\Big) + \end{aligned}$$

$$
\begin{aligned}
&+\frac{i}{6}\sqrt{a_0^{-1}}\Big(-\Sigma^m\gamma^u\partial_m A_{0u\bar{u}}+\Sigma^m\gamma^v\partial_m A_{o\bar{u}v}+\\
&+\gamma^{uv}\{\partial_u A_{0\bar{u}v}+\partial_v A_{0u\bar{u}}\}\Big)+\frac{i}{4}\sqrt{a_0^{-1}}K\Big]\,\epsilon=0\,,
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
D_{\bar{v}}\,\epsilon=\partial_{\bar{v}}\,\epsilon-\Big[&\frac{1}{4}\Big(g^{v\bar{v}}\partial_{\bar{v}}\,g_{v\bar{v}}+g^{u\bar{v}}\partial_{\bar{v}}\,g_{u\bar{v}}+g^{u\bar{u}}\partial_{\bar{u}}\,g_{u\bar{v}}+g^{v\bar{u}}\partial_{\bar{u}}\,g_{v\bar{v}}\Big)-\frac{1}{4}\,\Sigma^m\gamma^v\partial_m\,g_{v\bar{v}}-\\
&-\frac{1}{4}\,\Sigma^m\gamma^u\partial_m\,g_{u\bar{v}}-\frac{1}{4}\,\gamma^{uv}\left(\partial_u\,g_{v\bar{v}}-\partial_v\,g_{u\bar{v}}\right)-\\
&-\Sigma^m\gamma^u\frac{i}{12}\sqrt{a_0^{-1}}\Big(g^{\bar{u}v}R-g^{v\bar{v}}S\Big)+\Sigma^m\gamma^v\frac{i}{12}\sqrt{a_0^{-1}}\Big(g^{\bar{u}u}R-g^{u\bar{v}}S\Big)+\\
&+\frac{i}{6}\sqrt{a_0^{-1}}\Big(\Sigma^m\gamma^u\partial_m A_{0\bar{v}u}-\Sigma^m\gamma^v\partial_m A_{0v\bar{v}}-\\
&-\gamma^{uv}\{\partial_u A_{0v\bar{v}}+\partial_v A_{0\bar{v}u}\}\Big)+\frac{i\sqrt{a_0^{-1}}}{4}\,L\Big]\,\epsilon=0\,,
\end{aligned}
\tag{22}
$$

where

$$H=g^{u\bar{u}}\partial_m A_{u\bar{u}0}+g^{u\bar{v}}\partial_m A_{u\bar{v}0}+g^{v\bar{u}}\partial_m A_{v\bar{u}0}+g^{v\bar{v}}\partial_m A_{v\bar{v}0}\,, \tag{23}$$

$$I=g^{v\bar{v}}\partial_u A_{v\bar{v}0}-g^{v\bar{u}}\partial_u A_{\bar{u}v0}+g^{v\bar{v}}\partial_v A_{\bar{v}u0}-g^{\bar{u}v}\partial_v A_{u\bar{u}0}\,, \tag{24}$$

$$J=g^{u\bar{u}}\partial_u A_{\bar{u}v0}-g^{u\bar{v}}\partial_u A_{v\bar{v}0}+g^{u\bar{u}}\partial_v A_{u\bar{u}0}-g^{\bar{v}u}\partial_v A_{\bar{v}u0}\,, \tag{25}$$

$$P=\Big(g_{u\bar{u}}\,\partial_m A_{v\bar{v}0}-g_{\bar{u}v}\,\partial_m A_{u\bar{v}0}\Big)\,, \tag{26}$$

$$Q=\Big(g_{u\bar{u}}\,\partial_m A_{\bar{u}v0}+g_{\bar{u}v}\,\partial_m A_{u\bar{u}0}\Big)\,, \tag{27}$$

$$K=\Big(g^{v\bar{v}}\partial_{\bar{v}}\,A_{0\bar{u}v}-g^{\bar{v}u}\partial_{\bar{v}}\,A_{0u\bar{u}}+g^{v\bar{v}}\partial_{\bar{u}}\,A_{v\bar{v}0}-g^{\bar{v}u}\partial_{\bar{u}}\,A_{\bar{v}u0}\Big)\,, \tag{28}$$

$$R=\Big(g_{v\bar{v}}\,\partial_m A_{u\bar{u}0}-g_{\bar{v}u}\,\partial_m A_{v\bar{u}0}\Big)\,, \tag{29}$$

$$S=\Big(g_{v\bar{v}}\,\partial_m A_{\bar{v}u0}+g_{\bar{v}u}\,\partial_m A_{v\bar{v}0}\Big)\,, \tag{30}$$

$$L=\Big(g^{u\bar{u}}\partial_{\bar{u}}\,A_{u0\bar{v}}-g^{\bar{u}v}\partial_{\bar{u}}\,A_{0v\bar{v}}+g^{u\bar{u}}\partial_{\bar{v}}\,A_{u\bar{u}0}-g^{\bar{u}v}\partial_{\bar{v}}\,A_{\bar{u}v0}\Big)\,. \tag{31}$$

From eq. (17), equating the respective Γ terms we get,

$$\partial_0\epsilon=0\,, \tag{32}$$

$$\gamma_0\gamma^u\left(\frac{1}{4}\,\partial_u\ln a_0+\frac{i}{6}\sqrt{a_0^{-1}}\,I\right)=0\,, \tag{33}$$

$$\gamma_0\,\gamma^v\left(\frac{1}{4}\,\partial_v\ln a_0+\frac{i}{6}\,\sqrt{a_0^{-1}}\,J\right)=0\,, \tag{34}$$

$$\gamma_0\Sigma^m\left(-\frac{1}{4}\,\partial_m\ln a_0-\frac{i}{6}\,\sqrt{a_0^{-1}}H\right)=0\,. \tag{35}$$

Similarly equating the respective Γ terms in (18) we get:

$$\partial_n\,\epsilon = -\frac{i}{6}\sqrt{a_0^{-1}}\,H\epsilon\,, \tag{36}$$

$$\frac{1}{4}\,\partial_u \ln a_1 = \frac{i}{12}\sqrt{a_0^{-1}}\,I\,, \tag{37}$$

$$\frac{1}{4}\,\partial_v \ln a_1 = \frac{i}{12}\sqrt{a_0^{-1}}J\,, \tag{38}$$

$$\frac{1}{8}\,\partial_n \ln a_1 = \frac{i}{24}\sqrt{a_0^{-1}}H\,. \tag{39}$$

Comparing the above equations with eqs. (33), (34) and (35), we deduce

$$\partial_n\,\epsilon = \frac{1}{4}\,(\partial_n \ln a_0)\,\epsilon\,, \tag{40}$$

$$\partial_m \ln\sqrt{a_0^{-1}} = \partial_m \ln a_1\,, \tag{41}$$

$$\partial_\mu \ln\sqrt{a_0^{-1}} = \partial_\mu \ln a_1\,, \tag{42}$$

suggesting a relation

$$a_1 = \sqrt{a_0^{-1}}\,. \tag{43}$$

Clearly, we have not used the actual form of $g_{\mu\bar\nu}$ and $A_{0\mu\bar\nu}$ in deducing the relation between a_0 and a_1. We will see that the equations obtained by comparing Γ terms in (21), (22) will help us to determine three-form components and $g_{\mu\bar\nu}$. The set of equations we get from equating the respective γ terms in (21) are

$$\partial_{\bar u}\,\epsilon - \left[\frac{1}{4}\,(g^{u\bar u}\partial_{\bar u}\,g_{u\bar u} + g^{u\bar v}\partial_{\bar v}\,g_{u\bar u} + g^{v\bar v}\partial_{\bar v}\,g_{v\bar u} + g^{v\bar u}\partial_{\bar u}\,g_{v\bar u}) + \frac{i}{4}\sqrt{a_0^{-1}}K\right]\epsilon = 0\,, \tag{44}$$

$$\Sigma^m\gamma^v\left\{-\frac{1}{4}\,\partial_m\,g_{v\bar u} - \frac{i}{12}\sqrt{a_0^{-1}}\,(g^{\bar v u}P - g^{u\bar u}Q) + \frac{i}{6}\sqrt{a_0^{-1}}\,\partial_m\,A_{0\bar u v}\right\} = 0\,, \tag{45}$$

$$\Sigma^m\gamma^u\left\{-\frac{1}{4}\,\partial_m\,g_{u\bar u} + \frac{i}{12}\sqrt{a_0^{-1}}\,(g^{v\bar v}P - g^{\bar u v}Q) - \frac{i}{6}\sqrt{a_0^{-1}}\,\partial_m\,A_{0u\bar u}\right\} = 0\,, \tag{46}$$

$$\gamma^{uv}\left\{\frac{1}{4}\,(\partial_v\,g_{u\bar u} - \partial_u\,g_{v\bar u}) + \frac{i}{6}\sqrt{a_0^{-1}}\,(\partial_u\,A_{0\bar u v} + \partial_v\,A_{0u\bar u})\right\} = 0\,. \tag{47}$$

Similarly, we get the following set from eq. (22):

$$\partial_{\bar v}\,\epsilon - \left[\frac{1}{4}\Big(g^{v\bar v}\partial_{\bar v}g_{v\bar v} + g^{u\bar v}\partial_{\bar v}g_{u\bar v} + g^{u\bar u}\partial_{\bar u}g_{u\bar v} + g^{v\bar u}\partial_{\bar u}g_{v\bar v}\Big) + \frac{i}{4}\sqrt{a_0^{-1}}L\right]\epsilon = 0\,, \tag{48}$$

$$\Sigma^m\gamma^u\left\{-\frac{1}{4}\,\partial_m g_{u\bar v} - \frac{i}{12}\sqrt{a_0^{-1}}\,(g^{\bar u v}R - g^{v\bar v}S) + \frac{i}{6}\sqrt{a_0^{-1}}\,\partial_m\,A_{0\bar v u}\right\} = 0\,, \tag{49}$$

$$\Sigma^m\gamma^v\left\{-\frac{1}{4}\partial_m g_{v\bar v} + \frac{i}{12}\sqrt{a_0^{-1}}\,(g^{u\bar u}R - g^{\bar v u}S) - \frac{i}{6}\sqrt{a_0^{-1}}\,\partial_m\,A_{0v\bar v}\right\} = 0\,, \tag{50}$$

$$\gamma^{uv}\left\{-\frac{1}{4}\,(\partial_u\,g_{v\bar v} - \partial_v\,g_{u\bar v}) - \frac{i}{6}\sqrt{a_0^{-1}}\,(\partial_u\,A_{0v\bar v} + \partial_v\,A_{0\bar v u})\right\} = 0\,. \tag{51}$$

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There are at least *three* solutions solving eqs. (32)–(51).

(1) *For a sub-class of membranes satisfying* $\mathcal{N} = |\partial_u f|^2 + |\partial_v f|^2 = \text{const}$:

$$\begin{aligned} ds^2 &= H^{-2/3}(r,|f|)\left(-dt^2 + 2\,|du|^2 + 2\,|dv|^2 - \frac{2}{\mathcal{N}}\,|df|^2\right) + \\ &\quad + H^{1/3}(r,|f|)\left(\frac{2}{\mathcal{N}}\,|df|^2 + \sum_{i=5}^{10} dx_m^2\right), \end{aligned} \tag{52}$$

$$A = \frac{1}{2\mathcal{N}}\, i\, H^{-1/3}(r,|f|)\left\{-dt \wedge *\left(df \wedge d\bar{f}\right)\right\}, \tag{53}$$

$$\epsilon = \epsilon_0\, H^{-1/6}(r,|f|), \tag{54}$$

where the Hodge star operation $*$ is done on the $\mathbb{R}^2 \times \mathbb{T}^2$ space. The three-form potential in component form for the above metric, in the convention $\epsilon^{u\bar{u}v\bar{v}} = +1$, simplifies to:

$$\begin{aligned} A_{[0u\bar{u}]} &= i\,\frac{1}{\mathcal{N}}\, H^{-1}(r,|f|)|\partial_v f|^2, \qquad A_{[0u\bar{v}]} = -i\,\frac{1}{\mathcal{N}}\, H^{-1}(r,|f|)\,\partial_u f \partial_{\bar{v}} \bar{f}, \\ A_{[0v\bar{v}]} &= \frac{1}{\mathcal{N}}\, H^{-1}(r,|f|)|\partial_u f|^2, \qquad A_{[0v\bar{u}]} = -i\,\frac{1}{\mathcal{N}}\, H^{-1}(r,|f|)\,\partial_v f\, \partial_{\bar{u}} \bar{f}. \end{aligned} \tag{55}$$

This restricted class includes holomorphic curves $f = p\,u + qv$ representing planar membranes corresponding to (p,q) strings in IIB theory which preserve half supersymmetry. The membrane solution for $f = u$ and $f = v$ agrees with the results in ref. [8].

(2) *Intersecting* $M2 \perp M2$ *branes at a point* [*10*, *11*]:

$$\begin{aligned} ds^2 &= H_1^{1/3}(r)H_2^{1/3}(r)\Big\{ - H_1^{-1}(r)H_2^{-1}(r)\,dt^2 + 2\,H_1^{-1}(r)|du|^2 + \\ &\quad + 2\,H_2^{-1}(r)|dv|^2 + \sum_{i=5}^{10}(dx_m)^2\Big\}, \end{aligned} \tag{56}$$

$$A = i\,H_1^{-1} dt \wedge du \wedge d\bar{u} + i\,H_2^{-1} dt \wedge dv \wedge d\bar{v}, \tag{57}$$

$$\epsilon = \epsilon_0\, H_1^{-1/6} H_2^{-1/6}, \tag{58}$$

where ϵ_0 is a constant and $H_1(r)$, $H_2(r)$ are harmonic functions dependent only on the transverse coordinates common to both the branes. These solutions are meaningful only if U, V are compact coordinates with the charges being smeared over the branes.

(a) For a coordinate transformation $u = x_1 + i\,x_4\,; v = x_2 + i\,x_3$, the usual Kaluza-Klein reduction along x_3 on the above metric and T-duality along x_4 gives the following ten-dimensional string metric:

$$\begin{aligned} ds_{10}^2 &= -H_1^{-1/2}(r)H_2^{-1}(r)\,dt^2 + H_1^{-1/2}(r)\,dx_1^2 + \\ &\quad + H_1^{1/2}(r)H_2^{-1}(r)\,dx_2^2 + H_1^{1/2}(r)\sum_{m=4}^{10}(dx_m)^2. \end{aligned} \tag{59}$$

This represents delocalised solution for *orthogonal intersection of fundamental string along x_2 and D string along x_1.*

(b) For another coordinate transformation $u = (z_1\tau_2 - \tau_1 z_2), v = z_2$, where $z_1 = x_1 + i\,x_4,\ z_2 = x_2 + i\,x_3$, we obtain the following metric after dimensional reduction along x_3 and T-duality along x_4:

$$ds^2_{10} = B(r)\Bigg(- H_1^{-1/2}(r) H_2^{-1}(r)\, dt^2 + H_1^{-1/2}(r)(dx_1\tau_2 - dx_2\tau_1)^2 + \\ + H_2^{-1}(r) H_1^{1/2}(r)\, dx_2^2 + H_1^{1/2}(r)\,\tau_2^{-2} dx_4^2 + H_1^{1/2}(r)\sum_{m=5}^{10}(dx_m)^2\Bigg), \tag{60}$$

where $B(r) = \sqrt{1 + H_1^{-1}(r) H_2(r)\,\tau_1^2}$. This solution is the simplest planar network of F-strings and D-strings directed along $(0,1)$ and $(-\tau_2, \tau_1)$ in the (x_1, x_2) plane.

General planar network involving $[p_1, q_1], [p_2, q_2]$ strings can be similarly obtained using the coordinate transformation : $u = p_1(\tau_2\, z_1 - \tau_1 z_2) - q_1 z_2,\ v = q_2\, z_2 - p_2\,(\tau_2 z_1 - \tau_1 z_2)$.

The equivalence of delocalised orthogonal intersecting strings with general planar network of strings is expected because the delocalised solution in M-theory (58) has no information about the intersection point or the subspace containing the two M2-branes. *Hence eq. (58) also represent delocalised solutions for the membrane corresponding to general planar network of strings.*

In order to distinguish the intersecting membranes from curved membranes corresponding to three-string junction, we have to look for *fully or partially localised solutions.*

(3) *Arbitrary membranes including those corresponding to three-string junction*

$$ds^2 = -H^{-2/3} dt^2 + 2H^{-2/3}\, G_{\mu\bar\nu}(u, \bar u, v, \bar v, r)\, dx^\mu\, dx^{\bar\nu} + H^{1/3}\sum_{m=5}^{10}(dx_m)^2, \tag{61}$$

$$A = i\, H^{-1} G_{\mu\bar\nu}\, dx_0 \wedge dx_\mu \wedge dx_{\bar\nu}, \tag{62}$$

$$\epsilon = \epsilon_0\, H^{-1/6}, \tag{63}$$

where $G_{\mu\bar\nu}$ is Kahler and the function H is

$$H = G_{u\bar u}\, G_{v\bar v} - G_{u\bar v}\, G_{v\bar u}. \tag{64}$$

The metric in terms of the Kahler potential K is

$$G_{\mu\bar\nu} = \partial_\mu\, \partial_{\bar\nu}\, K. \tag{65}$$

The embedding of the membrane (1) in the metric can be seen by performing the following holomorphic coordinate transformation with unit jacobian:

$$(u, v) \longrightarrow (\alpha, \beta), \quad \text{where } \alpha = \sqrt{(u - \lambda_1)(v - \lambda_2)};\ \beta = \alpha \ln\frac{u - \lambda_1}{v - \lambda_2}. \tag{66}$$

The membrane surface in the new coordinate is $f = \alpha - \sqrt{\lambda_1 \lambda_2}$. This membrane is different from the intersecting membranes given by $u = \lambda_1; v = \lambda_2$. We hope to see this difference from partially or fully localised supergravity solution.

We are now left with the task of determining the form of K from the equations of motion for three-form gauge field in the presence of the curved membrane as the source:

$$\begin{aligned} \partial_M \left(\sqrt{-g} F^{MUVW}\right) + \frac{1}{1152} \left(\epsilon^{UVWMNOPQRST} F_{MNOP} F_{QRST}\right) = \\ = J^{UVW}(x) = \frac{1}{\sqrt{-g}} \frac{\delta S_{\text{membrane}}}{\delta_{A_{UVW}(x)}} \,, \end{aligned} \tag{67}$$

where the membrane action is

$$S_{\text{membrane}} = -T \int d^3\xi \left\{ \sqrt{-\det h} - \frac{1}{6}\, \epsilon^{abc}\, \partial_a X^M\, \partial_b X^N\, \partial_c X^P A_{MNP} \right\}. \tag{68}$$

Here $h_{ab} = \partial_a X^M \partial_b X^N G_{MN}$ is the induced metric on the membrane.

Choosing $\xi_0 = t$, $\xi_1 = \beta$, $\xi_2 = \bar{\beta}$, we obtain the three-form current for the holomorphic membrane f to be

$$J^{0\beta\bar{\beta}} = \frac{T}{\sqrt{-g}}\, \delta^2(f)\, \delta^6(x_m)\,. \tag{69}$$

Substituting the three-form potential and metric (61), (62) respecting one-fourth supersymmetry, we get

$$\partial_\mu\, \partial_{\bar{\nu}}\, (2\, H + \delta^{mn}\, \partial_m\, \partial_n K) = j_{\mu\bar{\nu}}\,, \tag{70}$$

where

$$j_{\mu\bar{\nu}} = \epsilon_{\mu\mu'}\, \epsilon_{\nu\nu'}\, J^{0\mu'\nu'} \sqrt{-\det g}\,. \tag{71}$$

The above non-linear equation cannot be solved analytically. Perturbative approach over Minkowskian backgrounds gives integral representation for K which is dependent on $j_{\mu\bar{\nu}}$ [12]. Since $j_{u\bar{u}} = \delta^2(u)\,\delta^6(r)$, $j_{v\bar{v}} = \delta^2(v)\,\delta^6(r)$ for the intersecting membranes is different from that of the curved membranes (69), the integrands of the integral representation for K are distinct.

In this approach [12], we expand the Kahler potential $K = \sum_n K^{(n)}$ and hence the metric $G_{m\bar{n}} = \sum_l G^{(l)}_{m\bar{n}}$. Minkowskian background implies that we take the zeroth-order $K^{(0)} = u\bar{u} + v\bar{v}$ so that $G^{(0)}_{m\bar{n}} = \delta_{m\bar{n}}$. Further, comparing n-th order terms in eq. (70) gives a set of differential equations. Using this set, we get a formal integral representation for $K^{(n)}$ involving the lower order metric components. However, the goal to obtain explicit closed form expression for localised or partially localised solution from the integral representation is still unsolved.

It has been shown, for certain classes of orthogonal intersections and holomorphic curves, that the perturbation theory breaks down when there are more than three

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transverse dimensions [12]. This breakdown of the perturbation theory suggests that no such fully localized solutions exist with asymptotically flat boundary conditions. However, perturbation theory suggests that such solutions do exist when there are less than three transverse dimensions. So, for sufficiently smeared versions of the sources, one expects that the solutions could be obtained numerically even if an exact analytic form cannot be found.

It is not clear at this stage whether perturbation theory over other backgrounds like planar membrane background would help in finding a closed form expression. We hope to pursue this issue in future.

The fully localised or partially localised supergravity solutions is also needed to understand the map of bulk parameters to boundary gauge theory parameters to prove AdS/CFT correspondence. Such near-horizon or AdS metric for intersecting branes [13, 14] and intersecting M5-branes [15] corresponding to NS5-D4 branes in IIA theory [16] has been obtained.

The procedure elaborated for one-fourth BPS states can be generalised to construct supergravity solution for other non-planar networks [17, 18]. The challenging problem of finding closed form expression for localised/partially localised solutions still remains.

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